STANDARD 6: MATHEMATICAL STRUCTURE/LOGIC

Students use both inductive and deductive reasoning as they make conjectures and test the validity of arguments.

READINESS (Kindergarten)

Students know and are able to do the following:

- **6M-R1. Sort and classify objects according to observable attributes**

- **6M-R2. Justify their answers and reasoning processes**

FOUNDATIONS (Grades 1-3)

Students know and are able to do all of the above and the following:

- **6M-F1. Recognize that numbers are used for different purposes in the world and a variety of mathematical notations represent these situations**

  PO 1. Formulate mathematical problems from everyday situations

- **6M-F2. Draw inductive and deductive conclusions about mathematics**

  PO 1. Extend a pattern using inductive reasoning (e.g., "What is the next number after 2, 4, 6, 8?")
  PO 2. Make a prediction based on existing information (e.g., "All the students in a 3rd grade class are under 10 years old. How old will the next new student probably be?")

- **6M-F3. Distinguish between relevant and irrelevant information**

  PO 1. Select the information necessary to solve a given problem

- **6M-F4. Interpret statements made with precise language of logic (e.g., *all, every, none, some, or, many*)**

  PO 1. Use words such as *all, every, none, some* and *many* to make reasonable conclusions about situations

ESSENTIALS (Grades 4-8)

Students know and are able to do all of the above and the following:

- **6M-E1. Use models to explain how ratios, proportions and percents can be used to solve problems and apply reasoning processes, such as spatial reasoning and reasoning with proportions and graphs**

  (Grades 4-5)

  Note: There are no POs for at this level

  (Grades 6-8)

  PO 1. Communicate how to solve problems involving ratios, proportions and percents using concrete and illustrative models

- **6M-E2. Construct, use and explain algorithmic procedures for computing and estimating with whole numbers, fractions, decimals and integers**

  (Grades 4-5)

  PO 1. Design a method with a series of defined steps for solving a problem; justify the method
  - A. whole numbers

  (Grades 6-8)

  PO 1. Design a method with a series of defined steps for solving a problem; justify the method
  - B. fractions, decimals and integers

- **6M-E3. Use *if . . . then* statements to construct simple valid arguments**

  (Grades 4-5)

  PO 1. Construct simple valid arguments using *if . . . then* statements based on
  - A. graphic organizers (e.g., Venn diagrams and pictures . . .)
  - B. geometric shapes

  (Grades 6-8)

  PO 1. Construct simple valid arguments using *if . . . then* statements based on
  - B. geometric shapes
  - C. proportional reasoning in probability
  - D. syllogism

  PO 2. Solve problems using deductive reasoning

PROFICIENCY (Grades 9-12)

Students know and are able to do all of the above and the following:

- **6M-P1. Use inductive and deductive logic to construct simple valid arguments**

Core – will be tested on AIMS

PO 2. Produce a valid conjecture using inductive reasoning by generalizing from a pattern of observations (e.g., if $10^1 = 10$, $10^2 = 100$, $10^3 = 1000$, make a conjecture)

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 1. Construct a simple informal deductive proof (e.g., write a proof of the statement: "Given an airline schedule with cities and flight times, you can fly from Bombay to Mexico City")

- **6M-P2. Determine the validity of arguments**

Core – will be tested on AIMS

PO 2. Draw a simple valid conclusion from a given *if . . . then* statement and a minor premise
PO 3. Distinguish valid arguments from invalid arguments
PO 4. List related *if . . . then* statements in logical order

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 1. Determine if the converse of a given statement is true or false
PO 6. Analyze assertions about everyday life by using principles of logic (e.g., examine the fallacies of advertising)

*Beyond Core**

PO 7. Recognize the difference between a statement verified by mathematical proof (i.e., a theorem) and one verified by empirical data (e.g., women score higher than men on vocabulary tests)
{PO 5 Deleted}

- **6M-P3. Formulate counterexamples and use indirect proof**

Core – will be tested on AIMS

PO 1. Construct a counterexample to show that a given invalid conjecture is false (e.g., Nina makes a conjecture that $x^3 > x^2$ for all values of x. Find a counterexample.)

*Beyond Core: Appropriate to be taught after a grounding in core instruction, but will not be tested on AIMS

- **6M-P4. Make and test conjectures**

Beyond Core

PO 1. Write an appropriate conjecture given a certain set of circumstances
PO 2. Test a conjecture by constructing a logical argument or a counterexample

- **6M-P5. Understand the logic of algebraic procedures**

Core – will be tested on AIMS

PO 1. Determine whether a given algebraic expression and a possible simplified form are equivalent (e.g., show that $(x + y)^2 = x^2 + y^2$ is invalid)

PO 2. Determine whether a given procedure for solving an equation is valid

DISTINCTION (Honors)

Students know and are able to do all of the above and the following:

- **6M-D1. Prove elementary theorems within various mathematical structures**

- **6M-D2. Develop an understanding of the nature and purpose of axiomatic systems**

- **6M-D3. Construct proofs for mathematical assertions, including indirect proofs and proofs by mathematical induction**